On the skin effect in symmetrically driven rf discharges

T. Mussenbrock[1], T. Hemke[1], D. Ziegler[1], R. P. Brinkmann[1], and M. Klick[2]

[1]*Institute for Theoretical Electrical Engineering, Center for Plasma Science and Technology, Ruhr University Bochum, D-44780 Bochum*
[2]*Plasmetrex GmbH, D-12459 Berlin, Germany*

In the field of low-temperature, low-pressure RF discharge modeling, the electrostatic approximation of Maxwell's equations is a very prominent assumption in order to simplify and ease simulation. However, with increasing driving frequencies and sizes of plasma reactors, e.g., for modern materials processing applications, the impact of electromagnetic effects, namely standing wave effects, edge effects, and the skin effect, is also increasing. For small discharges these effects are neglected from the outset. In our contribution we show that even for smaller discharges in the high plasma density regime electromagnetic effects play a significant role and thus the electrostatic approximation tends to lose its validity.

1 Introduction

In the field of low-temperature, low-pressure RF discharge modeling and simulation the electrostatic approximation of Maxwell's equations is one of the most favored assumption in order to simplify the very complex system of the underlying equations, which is needed for describing phenomena on the time scale of the applied radio frequency. In such electrostatic models electromagnetic effects are inherently neglected.

It has been shown that for modern plasma applications, e.g., in materials processing, electromagnetic effects play a crucial role, even for large area plasma sources, which become more and more important as wafer sizes reach the meter-scale. In addition to increasing sizes of the plasma source itself, the driving frequencies have been increased to 200 MHz, e.g., for dual frequency discharges [1,2,3]. With increasing driving frequencies and sizes of plasma reactors the impact of electromagnetic effects, namely the standing wave effect, the edge effect, and the skin effect, is also increasing. It is obvious that in such cases the electrostatic description of the discharge loses its validity.

In our contribution we show that even in a small, symmetric RF discharge (Fig. 1) in the high plasma density regime, driven at moderate 13.56-MHz-excitation, the skin effect is apparent. We show that the electrostatic approximation is thus not sufficient in order to describe high frequency effects.

Our contribution is organized as follows: We first collect the basic equations for the purpose of modeling and simulation of high frequency effects in capacitive rf discharges. The simplifications and assumptions are based on a scale analysis for an appropriate plasma regime. We then solve Maxwell's equations coupled to a linear cold plasma model for a symmetric parallel plate configuration neglecting the plasma boundary sheath and related nonlinear effects. We observe a non-negligible skin effect for the high density regime, which corresponds to a magnetic Reynolds number of $R_m \geq 1$.

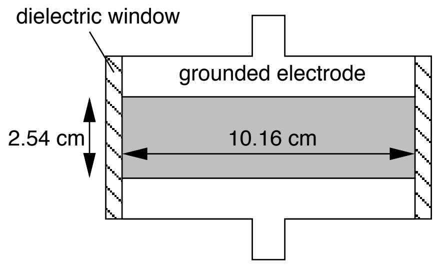

Fig. 1: Schematics of a small symmetric RF discharge.

2 Basic equations and typical scales

In order to describe high frequency effects, the electromagnetic fields in RF discharges can generally be calculated by the full system of Maxwell's equation coupled self-consistently to Boltzmann's equations for the single-particle distribution functions for each species. Since we are interested in the skin effect, we are able to drastically simplify Boltzmann's equation. We chose the driving frequency to lie between the ion plasma frequency and the electron plasma frequency. The ions are heavy particles, they react only to the phase-averaged electrical field and can thus be regarded as a stationary background, as well as the neutral gas. We are neglecting the effects of finite electron temperature and plasma chemistry (ionization/attachment). The high frequency current in the plasma is assumed to be a conduction current carried by the electrons alone. It is thus adapted to formulate the dynamics of the plasma by the so called cold plasma approximation, i.e., a generalized Ohm's law which takes into account electron acceleration by the electric field and momentum loss due to elastic collisions with neutrals of the background gas.

In order to facilitate the analysis with respect to length and time scales of the system, we employ a dimensionless notation. Based on a specific normalization basis we find a set of equations describing the skin effect:

$$\nabla \times \vec{B} = \vec{j}, \tag{1}$$

$$\nabla \times \vec{E} = -R_m \frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$\nabla \cdot \vec{B} = 0, \tag{3}$$

$$\nabla \cdot \vec{j} = 0, \tag{4}$$

$$\frac{\partial \vec{j}}{\partial t} = n_e \vec{E} - \nu \vec{j}. \tag{5}$$

$R_m = e^2 \bar{n} L^2 / m_e = L^2 / \lambda_{scf}$ denotes the magnetic Reynolds number. It is the square ratio of the typical reactor dimension L to the collisionless skin depth λ_{scf}. The magnetic Reynolds number indicates whether electromagnetic effects play a major role or the electrostatic approximation of Maxwell's equation is sufficient. $\nu = \nu_C / \omega_{rf}$ is the normalized electron collision rate for momentum transfer. We assume ν to be of the order of unity.

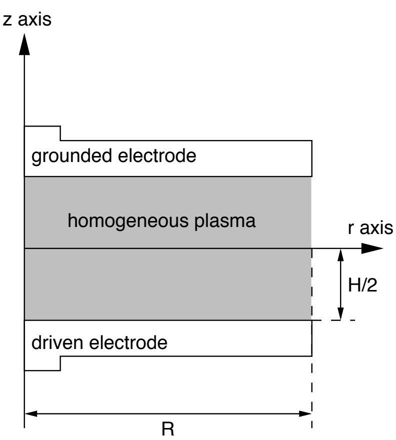

Fig. 2: Domain of modeling and simulation.

3 Skin effect

In order to study the skin effect we have to solve the given set of equations with appropriately chosen boundary conditions. Therefore, we concentrate on the domain which is indicated in Fig. 1 and Fig. 2, respectively. For simplicity we assume that the region between the electrodes is filled with a homogeneous plasma ($n_e = 1$). We restrict our analysis on the TM-mode in circular cylindrical coordinates. Hence, we assume that only an azimuthal magnetic field occurs. This means that for the electric field and the current density the azimuthal components vanish. As boundary conditions we assume that i) the radial component of the current density vanishes at $r = R$ and ii) a constant current density is impressed at $z = -H/2$.

After applying a Fourier transform we find for the azimuthal magnetic field a partial differential equation of the Helmholtz' type

$$\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} rB + \frac{\partial^2}{\partial z^2} B - \frac{iR_m}{i + \nu} B = 0, \tag{6}$$

with appropriate boundary conditions,

$$j_z\big|_{z=\pm H/2R} = \frac{1}{r} \frac{\partial}{\partial r} rB\big|_{z=\pm H/2R} = j_0, \tag{7}$$

$$j_r\big|_{r=1} = -\frac{\partial}{\partial z} B\big|_{r=1} = 0. \tag{8}$$

The problem to solve is an inhomogeneous boundary value problem, i.e., a homogeneous partial differential equation with inhomogeneous boundary conditions. This can be solved by the ansatz

$$B = \frac{j_0}{2} r + \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} b_{lm} J_1(\alpha_l r) \cos(\beta_m z) \tag{9}$$

The unknown coefficients can be found by a standard routine for solving a boundary value problem. The current density and the electric field can afterwards be calculated from the magnetic field.

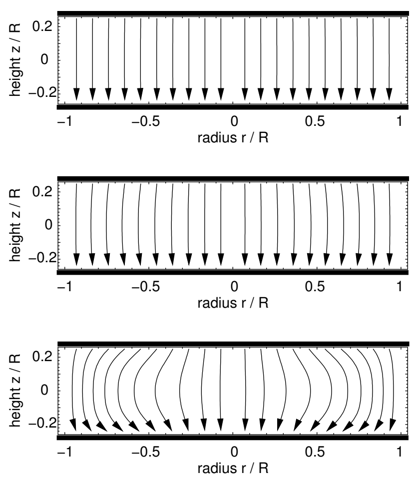

Fig. 3: Current density field lines for different magnetic Reynolds numbers. Above: $R_m = 9.13 \cdot 10^{-2}$ ($\bar{n} = 10^{15}\,\mathrm{m}^{-3}$), middle: $R_m = 0.913$ ($\bar{n} = 10^{16}\,\mathrm{m}^{-3}$) below: $R_m = 9.13$ ($\bar{n} = 10^{17}\,\mathrm{m}^{-3}$).

For different different magnetic Reynolds numbers, i.e., different plasma densities, the field lines of the current density are plotted. With increasing Reynolds numbers one observes bending of the current density field lines due to the growing influence of the skin effect.

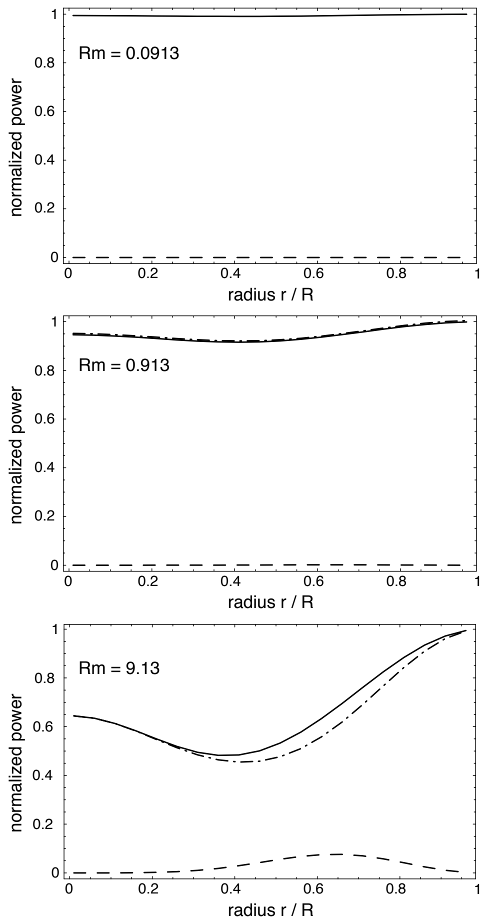

Fig. 4:Power coupled into the discharge for different magnetic Reynolds number. Total power P_{tot} (solid line), capacitive power P_{cap} (dash dotted line), inductive power P_{ind} (dashed line).

The skin effect becomes even more obvious, if the power coupled into the discharge is investigated. It is possible to define the powers coupled according to Lieberman idea as

$$P_{cap}(r) = \int_{-H/2}^{0} E_z^2(r,z)dz, \qquad (10)$$

$$P_{ind}(r) = \int_{-H/2}^{0} E_r^2(r,z)dz. \qquad (11)$$

The total power coupled to the discharge, $P_{tot} = P_{cap} + P_{ind}$, consisting of a capacitive and a inductive part, is for different Reynolds numbers depicted in Fig. 4. Within the high density regime an increasing influence of the skin effect on the total power coupled to the discharge is clearly observable.

4 Conclusions

Electromagnetic effects should not be neglected from the outset for modelling of small capacitively coupled rf discharges. In the regime of high plasma densities, which for the chosen reactor geometry corresponds to a magnetic Reynolds number larger than one, a significant skin effect is observable leading according to Lieberman to an inductive power coupling into the discharge. This cannot be explained on the basis of the electrostatic approximation and therefore requires a quasi-stationary approximation of Maxwell's equations.

5 Acknowledgement

The authors acknowledge the support by the Deutsche Forschungsgemeinschaft DFG via the Graduiertenkolleg 1051 “Non-equilibrium phenomena in low-temperature plasmas”.

6 References

[1] P. Chabert, J. Phys. D: Appl. Phys. 40, R63 (2007)
[2] M. A. Lieberman, J.-P. Booth, P. Chabert, J.-M. Rax, and M. M. Turner, Plasma Sources Sci. Technol. 11, 283 (2002)
[3] A. A. Howling, L. Sansonnes, J. Ballutaud, and C. Hollenstein, J. Appl. Phys. 96, 5429 (2004)