Numerical investigations of the integral of specific action of current for electrically exploded wires

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An MHD-approximation-based numerical simulation was used to investigate the electrical explosion of aluminum wires at current densities of 10^7–10^{10} A/cm^2 and explosion times of 10^{-6}–10^{-10} s. It has been shown that at current densities of 10^8–10^9 A/cm^2 the explosion mode changes from low-temperature to high-temperature, such that inertia forces become dominant which hinder the expansion of the wire. This transition is characterized by abrupt changes in thermodynamic parameters of the metal: the temperature and the energy delivered to the wire by the instant it explodes increase several fold. However, during this transition the specific action integral increases gradually, about three times, as the explosion characteristics (current density and explosion time) vary by two orders of magnitude.

1. Introduction

The electrical explosion of wires (EEW) has attracted the attention of researchers for years [1]. On the one hand, exploded wires are widely used in various engineering applications; on the other hand, they are of interest as objects of basic research. The EEW investigations are closely related to those on the pulsed breakdown in vacuum: during a vacuum breakdown there occurs an electrical explosion of the cathode surface, resulting in explosive electron emission [2]. The criterion for a pulsed breakdown in vacuum between a point cathode and a plane anode is [3]

$$j^2\tau_{br} = const\,, \qquad (1)$$

where j is the current density in the metal and τ_{br} is the time delay to the explosion of the field emitter. Relation (1) is closely associated with an EEW parameter: the integral of specific action of current, h, which is used in describing EEW by similarity methods [4]. For the conditions where the conductivity of the metal, σ, depends only on the energy density ε_w delivered to the wire, the specific action integral has the form

$$h = \int_{t_0}^{t} j^2 dt = \int_{\varepsilon_0}^{\varepsilon_w} \sigma(\varepsilon_w) d\varepsilon_w\,. \qquad (2)$$

When describing EEW, two forms of the specific action integral are generally used [3]: the specific action from room temperature to melting, h_1, and the specific action from melting to explosion, h_2. While the specific action h_1 is, with good accuracy, a constant for a given material, the specific action h_2 depends on the current density in the conductor. In this work, we computed the total integral $h = h_1+h_2$.

Experimentally [3], specific actions have been determined for wires of diameter up to 10 µm at current densities of 10^7–10^8 A/cm^2. However, from the viewpoint of investigation of the explosive electron emission during a vacuum breakdown, of greatest interest are the explosions of wires about 1 µm in diameter that occur at current densities of ~10^9 A/cm^2 with explosion times of 10^{-10}–10^{-9} s. Now the investigations of EEW in these modes can hardly be performed experimentally; however, they can be carried out by numerical simulation methods.

2. The MHD model and calculation results

To describe the processes that are involved in EEW, a magnetohydrodynamic (MHD) approximation is used. Numerical calculations in terms of this approximation demand a knowledge of the equations of state of the material in a wide range of thermodynamic parameters and of the transport coefficients, most important of which is electrical conductivity. The simulation of EEW was carried out in terms of the one-temperature MHD approximation that was numerically realized in the EXWIRE one-dimensional MHD code [5]. For the metal, wide-band semiempirical equations of state were used [6] which take into account the effects of high-temperature melting and evaporation. The conductivity of aluminum was taken from tables [7] compiled by M. Desijarleis at Sandia National Labs, USA.

It was supposed that the wire carries a linearly increasing current:

$$I(t) = \pi r_0^2 \left(\frac{dj}{dt}\right) t\,, \qquad (3)$$

where r_0 is the initial radius of the wire and $\left(\frac{dj}{dt}\right)$ is the rate of rise of current density. In each calculation variant, $\left(\frac{dj}{dt}\right)$ was a constant. By varying its value from $2.5\cdot10^{14}$ to 10^{20} A/cm$^2\cdot$s, we could simulate EEW modes with explosion times from some submicroseconds to less than 100 ps.

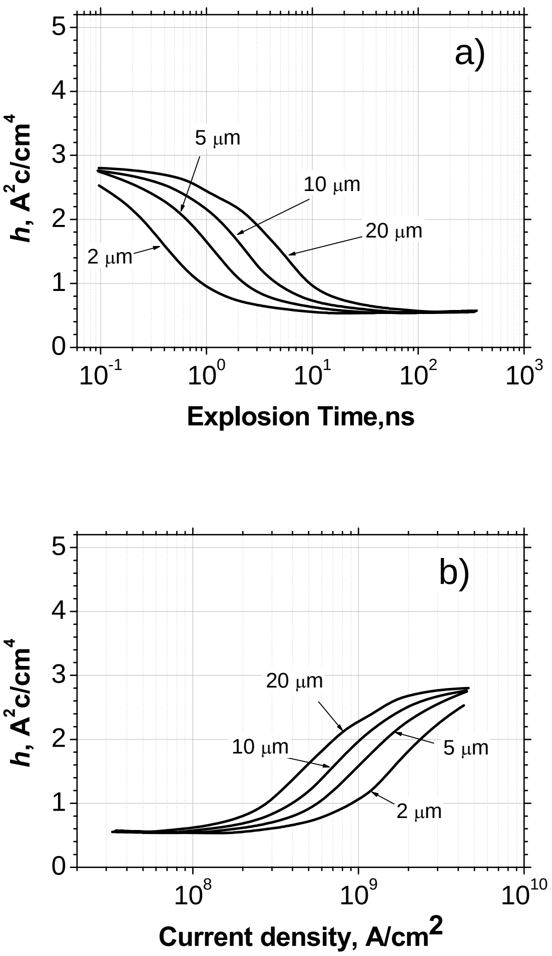

Fig. 1. Specific action integrals versus explosion time (a) and current density (b) for the explosion of wires of diameter 2, 5, 10, and 20 μm.

The explosion of aluminum wires in water and in vacuum was numerically simulated; the wire diameter was varied from 2 to 20 μm. Figure 1 presents the specific action integrals versus explosion time (Fig. 1a) and current density (Fig. 1b) for EEW in vacuum. As can be seen from Fig.1a, for explosion times longer than 10 ns (or for current densities <10^8 A/cm^2) the action integral h weakly depends on both the explosion time (or on the current density) and the wire dimensions. The specific action integrals start increasing at current densities greater than 10^8–10^9 A/cm^2 and, accordingly, at explosion times shorter than 10 ns (Fig. 1). The point at which the increase of an action integral begins depends on the diameter of the exploding wire (for wires of greater diameter the action integral starts increasing at lower current densities and at longer explosion times) and does not depend on the properties of the medium in which the explosion occurs (for wires of the same diameter the action integrals start increasing at the same explosion times for both the EEW in water and in the EEW in vacuum). The increase in action integrals is accompanied by variations of the thermodynamic parameters of the metal (the energy delivered to the wire, its temperature and density) .
Figure 2 presents, for a wire of diameter 5 μm, the thermodynamic parameters of the metal at the explosion time: the temperature T, the energy delivered to the wire, E_w, related to the sublimation energy E_{sub} (the energy required to completely evaporate the metal), and the specific density ρ/ρ_0 (the density of the metal related to its density under normal conditions (for aluminum $\rho_0 = 2.71$ g/cm^3)).

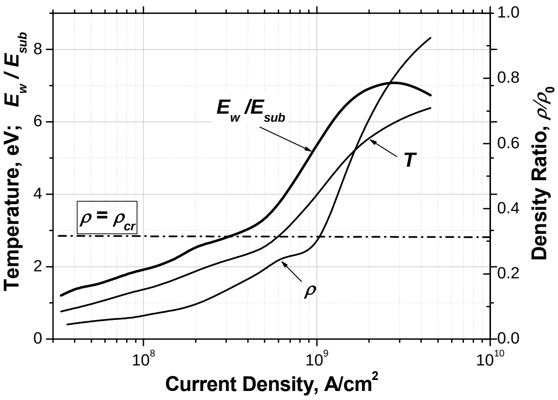

Fig. 2. Thermodynamic parameters of the metal of a wire versus current density at the explosion time, τ_{ex}.

As can be seen from Fig. 2, the increase in action integrals is accompanied by an increase in metal density at the explosion time τ_{ex}, by an increase in temperature to 5–6 eV and, as a consequence of the latter, by an increase in energy delivered to the wire to several sublimation energies. The thermodynamic parameters increase rather quickly, and this suggests that the increase in current density results in the transition of EEW from a low-temperature to a high-temperature mode, in which the energy delivered to the wire is several times greater than the energy that should be expended to completely evaporate the metal.

3. Results and discussion

From experimental studies of EEW it is well known that the higher the rate of current rise, the greater the energy delivered to the wire during the explosion. At high rate of current rise, the energy that can be delivered to the wire is several times greater than the sublimation energy for the metal, and this is true not only for the explosion in a gas medium [8], but also for the explosion in vacuum [9]. Two different suppositions have been made about the reasons for this behavior of the wire material. The first one is that the magnetic pressure plays the dominant part in this process and the second one is that the inertia forces prevent the expansion of the wire to considerable dimensions within short times. The second supposition has been supported most convincingly [10], and it has also been confirmed by our calculations.

Let us estimate the EEW parameters at which the effect of inertia forces is substantial. It is well known that at moderate current densities the explosion of a wire occurs when the metal density is close to the density at the critical point of the metal phase diagram where the liquid, gas-plasma, and two-phase regions meet. This can also be seen in Fig. 2 where the density at the critical point, ρ_{cr}, is shown by the dash-dotted line. In general, the critical density ρ_{cr} is less than the normal density ρ_0 by a factor of 3–5; therefore, the wire expands approximately twice in radius by the onset of explosion. Thus, the explosion time can be estimated as

$$\tau_{ex} \approx \frac{r_0}{v} \ , \qquad (4)$$

where v is the velocity of expansion of the wire. An estimate of this velocity can be obtained from the law of conservation of momentum if we take the values of the thermodynamic parameters equal to their values at the critical point:

$$\rho_{cr} \frac{v}{\tau_{ex}} \approx \frac{p_{cr}}{r_0} , \qquad (5)$$

where p_{cr} is the pressure at the critical point. Then (4) and (5) yield a condition for the transition from the low-temperature to the high-temperature explosion mode, which relates the wire dimension to the explosion time:

$$r_0 < \tau_{ex} \sqrt{\frac{p_{cr}}{\rho_{cr}}} \ . \qquad (6)$$

For aluminum, we have p_{cr} = 4.45 kbar, ρ_{cr} = 0.855 g/cm^3, and T_{cr} = 0.55 eV. Substituting p_{cr} and ρ_{cr} in (6), we obtain that for wires of diameter 2, 5, 10, and 20 µm the explosion times at which the low-temperature EEW changes into the high-temperature EEW are 1.4, 3.5, 7, and 14 ns, respectively. These times fit well to the calculated relations presented in Fig. 1a.

Let us consider the position of the curve that corresponds to expression (6) in the diagram of EEW modes (Fig. 3). This diagram has been constructed in accordance with the classification of EEW modes put forward by Chace and Levine [1]. It is based on the proportion between the characteristic times of various processes which occur during EEW. The basic times that can be used to characterize an EEW are: the time for which the material loses its metallic conduction (τ_{input}), the time for which MHD instabilities develop (τ_{inst}), and the current skinning time (τ_{skin}).

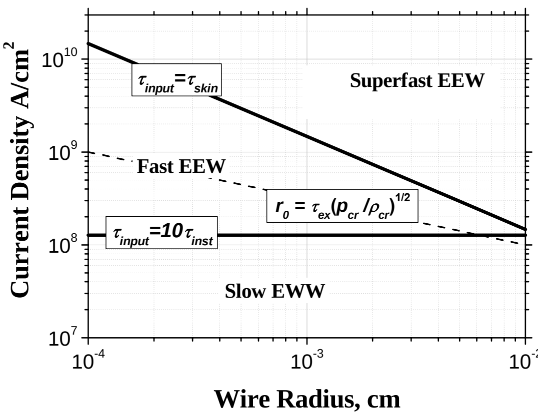

Fig. 3. Diagram of EEW modes for aluminum conductors.

The characteristic time of loss of metallic conduction is determined by the ratio of the sublimation energy to the power of the energy delivery due to Joule heating:

$$\tau_{\text{input}} = \frac{\rho \Lambda_i}{m_i} \frac{\sigma}{j^2} \ , \qquad (7)$$

where Λ_i is the sublimation energy per atom, ρ is the density of the metal, m_i is the atomic mass, and σ is the conductivity of the metal.

The characteristic time for the neck-type MHD instability (mode with m = 0) is determined as

$$\tau_{\text{inst}} = \frac{r_0}{c_A} = \frac{r_0\sqrt{4\pi\rho}}{B_\varphi} = \frac{c\sqrt{\rho}}{\sqrt{\pi}\, j} \qquad (8)$$

where B_φ is the magnetic field strength, $c_A = \frac{B_\varphi}{\sqrt{4\pi\rho}}$ is the Alfven velocity, and c is the velocity of light in vacuum.

The skinning time is determined by the diffusion of the magnetic field and depends on the wire dimension and conductivity:

$$\tau_{\text{skin}} = \frac{4\pi r_0^2 \sigma}{c^2}. \quad (9)$$

If the characteristic time of development of MHD instabilities in an EEW is much less than the time of loss of metallic conduction ($\tau_{\text{inst}} \ll \tau_{\text{input}}$), a slow mode is realized; otherwise a fast EEW mode takes place. From expressions (7) and (8), for a tenfold difference between these times, we obtain that for the fast mode to be realized the current density must satisfy the condition

$$j > \frac{\sqrt{\pi\rho}\Lambda_i \sigma}{10 m_i c} \quad (10)$$

Besides the slow and the fast EEW mode, a superfast mode is generally considered, such that the magnetic field has no time to penetrate deep into the bulk of the wire; that is, skinning of the current occurs. This mode is realized if the characteristic skinning time is longer than the characteristic time of loss of metallic conduction ($\tau_{\text{skin}} > \tau_{\text{input}}$). For this case, from expressions (7) and (9) we obtain the following condition for the current density:

$$j > \frac{c}{2r_0}\sqrt{\frac{\rho\Lambda_i}{\pi m_i}}. \quad (11)$$

As can be seen from Fig. 3, for aluminum, the fast EEW mode is realized at current densities greater than ~10^8 A/cm^2; for wires of micrometer size, the superfast mode takes place at current densities above ~10^{10} A/cm^2.

Relating the explosion time τ_{ex} to the current density in the wire via the specific action integral as $h \approx j^2 \tau_{ex}$ and substituting this in (6), we obtain

$$j < \sqrt{\frac{h}{r_0}\left(\frac{p_{cr}}{\rho_{cr}}\right)^{1/2}}. \quad (12)$$

If relation (12) holds, inertia forces do not influence the expansion of the exploding wire and the low-temperature explosion mode is realized. In the reverse case, the dominant part in the EEW is played by inertia forces which hinder the expansion of the wire, and the energy delivered to the wire material during the explosion is sufficient to heat the material to several electron-volts. The position of the curve fitting expression (12) is shown in Fig. 3 by the dash-dotted line. As can be seen from Fig. 3, this line divides the fast EEW mode region into two subregions in one of which a low-temperature explosion is realized, while in the other a high-temperature explosion takes place.

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4. References

[1] Chace W.G., Levine M.A.// J.Appl.Phys., 1960,V.31, N 7, p.1298.

[2] Mesyats G.A.., Proskurovskiy D.I.// Pis'ma v JETF (in Russian), 1971, V.13, N.1, p.7.

[3] Mesyats G.A..// Ectony v vacuumnom razriade-proboi, iskra,duga.(in Russian) – M.:Nauka, 2000, 424 p.

[4] Sedov L.I. Metody podobia I razmernosti v mehanike.(in Russian) – M.:Nauka, 1965, 447 p.

[5] V. I. Oreshkin, R. B. Baksht, A. Yu. Labetsky et.al.// Technical Physics, 2004, V.49, N.7, p.843.

[6] Tkachenko S.I., Khishenko K.V., Vorob'ev V.S. et al // TVT (in Russian), 2001, V. 39, p.728.

[7] Desjarlais M.P. // Contrib. Plasma Phys., 2001, V.41, N.2, p.267.

[8] Sedoy V.S., Mesyats G.A., Oreshkin V.I. et al. // IEEE Transactions on Plasma Science, 1999, V.27, N.4, p.845.

[9] Sarkisov G.S., Rosental S.E., Cochrane K.R. et.al.// Phys.Rev.E, 2005, V.71, p.046404.

[10] Kotov Yu.A., Samatov O.M., Sedoy V.S. et.al.// Megagauss Field and Pulsed Power Systems, MG-V, Nova Science Publishers, NY, 1990, p.497.