The use of a direct numerical solution of the radiative transfer equation to improve the simulation of streamer discharges.

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It is well known that the production of secondary electron by photoionisation and photoemission is very often necessary to explain the propagation of streamers. Classical integral methods which are generally used to calculate the photoionisation source term are computationally very expensive. It was shown recently [1], that the treatment of photo ionisation using an improved Eddington approximation, allows very fast and accurate calculations for absorption coefficient values higher than 50 cm^{-1}. The objective of this work is to employ a direct numerical method of solution of the radiative transfer equation. In this case, there is no limitation on the value of the absorption coefficient. The validity of this method is checked with a Gaussian emission source and for a double-headed streamer in a two-dimensional axisymmetric cylindrical geometry, in air, at atmospheric pressure.

1. Introduction

Discharges at high pressure usually consist of thin plasma filaments (called streamers) characterised by a high concentration of charged species (electrons and ions). Molecules and atoms excited by electron collisions may produce photons with energy sufficiently high to ionise other species or emit secondary electrons from electrodes These photons are, in many situations, the main mechanism of positive streamer propagation.

In most works, calculation of the photoionisation source term is made by using an integral form of the photon transport equation [4]. Using this approach, as a huge number of quadratures have to be done over the complete volume of the discharge, the calculations take a very long time. A new approach was recently proposed [1] based on an approximation of the differential form of the photon transport equation. This approach known as Eddington approximation is currently used in radiative transfer theory. An improved Eddington approximation was presented in [1].

Both approximations only work for situations in which the photons have a relatively short path length. In [1], it was shown that the lowest limit for the absorption coefficient to be used is 50 cm^{-1}. To extend this limit, we propose to solve the photon transport equation without any simplifications. With this technique, there is no limitation on the value of the absorption coefficient to be used.

2. Model formulation

2.1 Charged particle transport

The drift-diffusion equations which describe the space and time variations of the density of charged particles take the following form (for electrons and positive ions):

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}.n_e\vec{w}_e - \vec{\nabla}.(\vec{\vec{D}}_e\vec{\nabla}.n_e) = S_{ph} + S_e^+ - S_e^- \quad (1)$$

$$\frac{\partial n_p}{\partial t} + \vec{\nabla}.n_p\vec{w}_p - \vec{\nabla}.(\vec{\vec{D}}_p\vec{\nabla}.n_p) = S_{ph} + S_p^+ - S_p^- \quad (2)$$

In above equations, subscripts '*e*' and '*p*' refer to electrons and positive ions, respectively. n_i is the density of species *i*, $\vec{w}_i = \mu_i^*\vec{E}$ ($\vec{E}$ being the electric field) is the drift velocity of species *i*, $\vec{\vec{D}}_i$ and μ_i^* are the corresponding diffusion tensor and mobility. S^+ and S^- are the rates of production and loss of charged particles. S_{ph} is the rate of electron-ion pair production due to photoionisation in the gas. This term is directly related to the isotropic part $\psi_0(\vec{r},t)$ of the photon distribution function $\psi(\vec{r},\vec{\Omega},t)$ at position $\vec{r}$ in direction $\vec{\Omega}$ and at time t defined by (assuming a monochromatic radiation):

$$S_{ph}(\vec{r},t) = c\sigma^{ph}\int_{\Omega} d\Omega\,\psi(\vec{r},\vec{\Omega},t) = c\sigma^{ph}\psi_0(\vec{r},t)$$

where c is the speed of light and σ^{ph} is the photoionisation coefficient which characterizes the photoionisation efficiency.

Note that Poisson equation is solved together with equations (1) and (2).

2.2 Photon transport

If we take into account that the speed of light is much higher than the typical electron velocity, we can assume that the photons are in a stationary state. The radiative transfer equation can then be written:

$$\vec{\Omega}.\vec{\nabla}\psi(\vec{r},\vec{\Omega}) + \sigma\psi(\vec{r},\vec{\Omega}) = Q(\vec{r}) \quad (3)$$

In the above equation σ and $Q(\vec{r})$ are the absorption coefficient and the emission source term, respectively. In the following we will restrict our investigation to a two-dimensional axisymmetric discharge. Equation (3) above can then be written under the classical conservative form:

$$\frac{\mu}{r}\frac{\partial}{\partial r}[r\psi(z,r,\theta,\varphi)] + \xi\frac{\partial \psi}{\partial z}(z,r,\theta,\varphi) - \frac{1}{r}\frac{\partial}{\partial \varphi}[\eta\psi(z,r,\theta,\varphi)] + \sigma\psi(z,r,\theta,\varphi) = Q(z,r) \quad (4)$$

where θ and φ are polar and azimuthal angles, z and r are the longitudinal and radial positions. Variables $\mu\,(=\sin\theta\cos\varphi)$, $\eta\,(=\sin\theta\sin\varphi)$ and $\xi\,(=\cos\theta)$ are direction cosines defined in Figure 1 below:

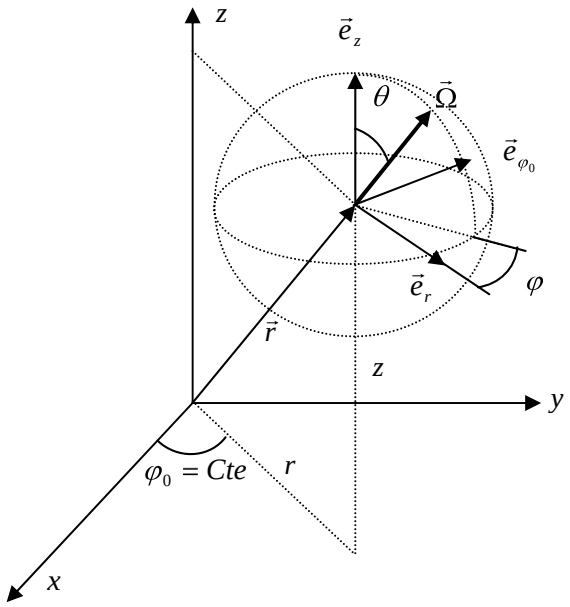

Figure 1 Cylindrical coordinates for the radiative transfer equation.

Equation (4) is solved by using a finite volume method (extension of the S_N method). Details can be found in [2] and [3]. Basically, the principle of the finite volume method is to build a discrete form of (4) by carrying out a quadrature over a volume element of the real space (i. e. $d^3r = rdrdz$) and over a volume element of the unit sphere (i. e. $d\Omega = \sin\theta d\theta d\varphi$). Doing that, we obtain a recurrence relationship which depends upon the values of the photon distribution function at the input and output faces of the volume element and upon the value at the centre. A specific assumption of the variation of the distribution function inside the volume element allows to connect the value at the centre with the value at the input face [3]. If the value at the input face is known, it is then possible to obtain the value at the centre and then (using the interpolation relationship) the value at the output face. Applying the same procedure for every volume element, the distribution function is obtained step by step for the whole volume. The marching direction depends on the sign of the direction cosines (from the outside of the cylinder to the centre, from the bottom to the top, etc.).

For sufficiently high values of σ, it is possible to assume that the distribution function weakly depends on direction $\vec{\Omega}$. In these conditions, we obtain a simple equation giving the isotropic part of the distribution function, which is the Eddington approximation. We have:

$$-\vec{\nabla}\left(\frac{1}{3\sigma}\vec{\nabla}\psi_0(z,r)\right) + \sigma\psi_0(z,r) = 4\pi Q(z,r) \quad (5)$$

Using a slightly less restrictive assumption for the dependence of $\psi(\vec{r},\vec{\Omega})$ with $\vec{\Omega}$ [1], we can obtain an improved Eddington approximation (called SP_3). In this case, two equations similar to (5) have to be solved simultaneously.

The main interest of these approximations is that the equations obtained are simpler than the direct numerical approximation of (4). In fact, they are very similar to Poisson's equation and can be solved by using the same numerical routine as used for electric potential computation. However, as we will see in the following, their validity is limited.

3. Results and discussion

3.1. Gaussian emission source

Our objective is now to check the validity of the direct numerical solution of the radiative transfer equation. To do that, we used a Gaussian emission source term $Q(\vec{r})$ centred on the symmetry axis of the cylindrical computational domain and given by:

$$Q(z,r) = \exp\left[-\left(\frac{z-z_0}{\sigma_z}\right)^2 - \left(\frac{r}{\sigma_r}\right)^2\right] \quad (6)$$

where $z_0 = 0.05\,cm$ and $\sigma_z = \sigma_r = 0.01\,cm$. Integration over space of equation (3) gives:

$$\psi_0(z,r) = \int_{vol} d^3r' \frac{Q(z,r)}{R^2}\exp(-\sigma R) \quad (7)$$

where $R = |\vec{r} - \vec{r}'|$ is the distance between the emission and observation points which has to be written in cylindrical coordinates. Equation (7) above is the usual integral form of equation (3). Using Gaussian quadratures, it is easy to calculate the isotropic distribution function $\psi_0(\vec{r})$ given by (7). Figure (2) gives the results obtained using direct and integral method. Calculations were made for absorption coefficients of 10 and 100 cm^{-1} respectively.

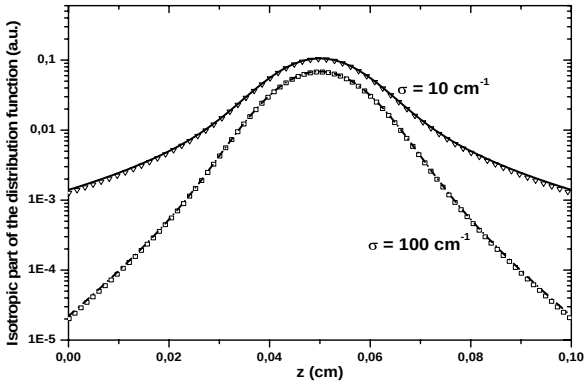

Figure 2 Isotropic part of the distribution, direct resolution (lines); integral method (symbols)

We see that there is a very good agreement between the two methods whatever the value of σ. It is clear now that the validity of the direct method is not dependent of the order of magnitude of the absorption coefficient σ.

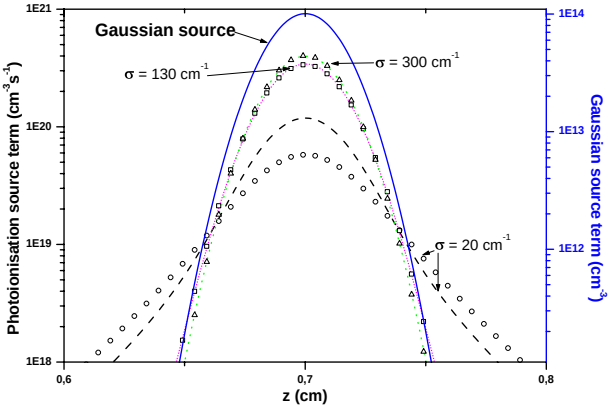

Figure 3 Photo ionisation source term for initial Gaussian electron density; direct resolution (lines); Eddington approximation (symbols)

3.2. Double-headed streamers in air

In this section, we report and compare modelling results on a double-headed streamer developing in air at pressure of 760 Torr obtained with the three different photo ionisation models. Two remote electrodes establish a uniform Laplacian field $E_0=4.8\times10^6$ V/m. All results presented in this paper are obtained assuming air neutral density $N_0=2.6\times10^{19}$ cm^{-3}, and therefore E_0/N_0 = 178.6 Td (1 Td=10^{-17} V cm^2). Under the influence of this applied field, a double-headed streamer is launched by placing a neutral plasma cloud in the simulation domain. The initial plasma cloud has a Gaussian distribution in space similar to that used in the previous section.

The centre of the Gaussian distribution is located in the middle of the simulation domain, at z_0 = 0.7 cm, and it is assumed that $\sigma_z = \sigma_r$ = 0.02 cm and $n_0=10^{14}$ cm^{-3}. The size of the computational domain is 1.4 × 0.125 cm. The computational grid is uniform in both radial and axial directions with 1681 and 151 cell points respectively. Further details may be found in [4].

Figure 3 shows the results obtained for the source term S_{ph}, calculated at the initial time, with the direct method and the Eddington approximation for three different values of the absorption coefficient: 300, 130 and 20 cm^{-1}. We note a perfect agreement for 300 cm^{-1} and minor differences for 130 cm^{-1}. As expected for 20 cm^{-1}, the Eddington model underestimates the maximum of the photoinisation source term close the centre of the Gaussian source and overestimates it for large values of z.

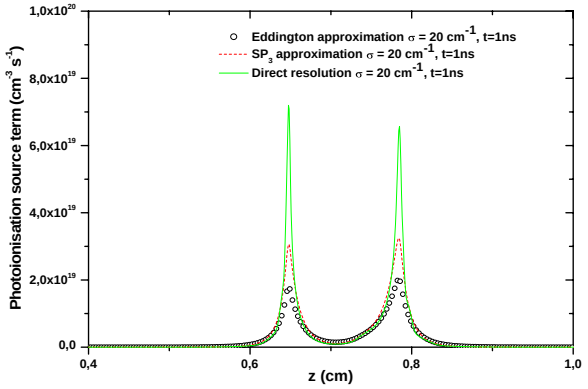

Figure 4 Photoionisation source term calculated at 1 ns with an absorption coefficient equals to 20 cm^{-1}. Calculations were made with the Eddington approximation, the SP_3 approximation and the direct method.

In Figure 4 above, the photoionisation source term was calculated at 1 ns for an absorption coefficient of 20 cm^{-1} and by all three methods (Eddington and SP_3 approximations, direct method). Significant discrepancies are observed between the three methods. We note that the use of the SP3 approximation allows to improve significantly the agreement with the direct method in comparison to the Eddington approximation. The photoionisation source term is smaller for the two approximate methods. This result is not surprising. Indeed, an approximation based upon a second order partial

differential equation involves a certain amount of smoothing and then a decrease of the maximum of the ionization intensity.

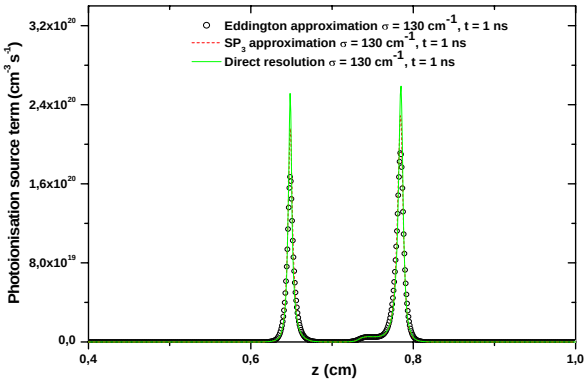

Figure 5 Same conditions as in Figure 4 but with an absorption coefficient equal to 130 cm^{-1}.

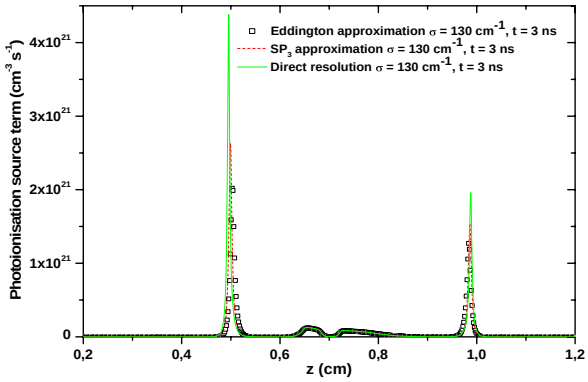

Figure 6 Same conditions as in Figure 5 but calculations are made at 3 ns.

Figure 5 and 6 give the same results for absorption coefficient of 130 cm^{-1} at time moments of 1 and 3 ns. At 1 ns, we can see that SP_3 and direct method are in very good agreement. However discrepancies occur at 3 ns between the three methods. These discrepancies are due to the fast spatial variation of the source term. Note that there is no difference between the two peaks, between 0.6 and 0.8 cm, where the variation is small.

It is interesting now to check the electron density profiles induced by the use of these methods. Figure 7 gives the space variation of electron density corresponding to an absorption coefficient of 130 cm^{-1} for seven successive time moments ranging from 0 ns to 1.2 ns. The comparisons are made between SP_3 approximation and the direct method. It can be seen that the results are in good agreement. This indicates that differences in intensity of ionisation sources do not necessary induce the corresponding differences in variation of the electron density.

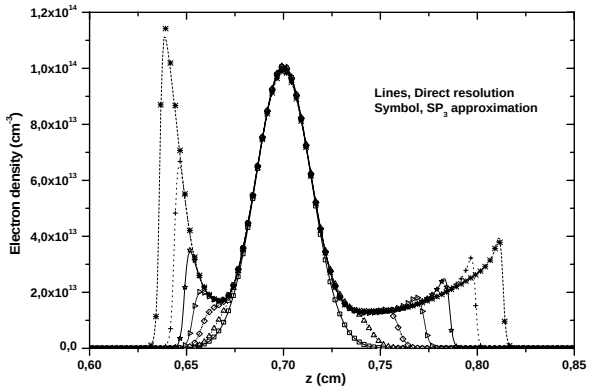

Figure 7 Space variation of electron density for different times ranging from 0 to 1.2 ns and for an absorption coefficient equal to 130 cm^{-1}.

4. Conclusion

Calculations made in this paper show the effectiveness of the coupling of a direct method of solution of the radiative transfer equation to usual equations employed in the numerical modelling of streamers. The accuracy of the direct method does not dependent on the value of the absorption coefficient, which allows to use this approach for a wide range of absorption coefficients σ starting from zero. Furthermore as it is not an iterative method the calculation time is of the same order of magnitude as the calculation time corresponding for Eddington or SP_3 approximation.

5. References

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