Limiting Behaviour of a Magnetised Presheath

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We present a one dimensional fluid model of a plasma in contact with an absorbing, planar wall under the influence of a magnetic field at an oblique angle to the wall. The resulting equations are integrated numerically to provide presheath profiles as well as trends in the presheath parameters for different conditions. Two limiting regimes are identified - the collision dominated and the magnetised presheath. The latter is found to be divided into two layers, the **B**-aligned presheath (BAP) and a Chodura layer. A simple analytic expression for the BAP width in terms of the collision mean free path and the field angle is determined for high Hall parameters. It is found that the width of the Chodura layer is a more complicated function of the field angle.

1. Introduction

The boundary of a magnetised plasma is of great interest to several fields ranging from material processing to magnetic confinement fusion. The scrape-off layer of a tokamak, for example, has low density and strong magnetic fields. Further, the angle of incidence of the field onto the divertor spans nearly the whole range from perpendicular to grazing. In this paper, we investigate the presheath structure in the high magnetisation limit and attempt to explain it in terms of two regions with characteristic ion flows. The model is based on an ion fluid description proposed by Riemann [1].

2. The Model

2.1. Equations and Geometry

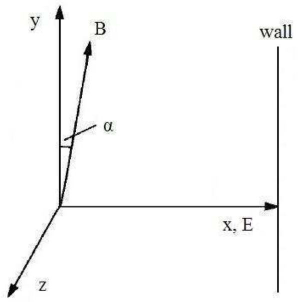

Fig 1: Geometry under consideration.

We consider the one-dimensional case under the influence of a uniform magnetic field at some angle α to the wall. Directions are chosen such that the field lies entirely in the x-y plane, where x is perpendicular to the wall, y is parallel to it and z is chosen to complete the right handed basis set. This implies that the electric field of the wall has only an x component and that the ion **E**x**B** drift is purely in the z direction. We only take account of the quasi-neutral presheath in this work, which is defined as the region between the undisturbed bulk plasma and the sheath edge, where the Bohm criterion must be fulfilled [2]. This requires that the space-charge sheath, which has a scale length of the order of a Debye length, is very small compared to the scale length of the presheath. It is assumed that the electrons are Boltzmann distributed and that the ion pressure obeys an ideal gas law. We also assume that the ions and electrons are isothermal across the presheath with temperatures T_i and T_e, respectively. The two equations which determine the ion fluid velocities and the potential are ion continuity and momentum (eqn. 1).

$$m_i(\vec{v}.\nabla)\vec{v} = e(\vec{E} + \vec{v}\times\vec{B}) - \frac{1}{n_i}\nabla p_i - m_i \nu_t \vec{v}$$

$$\nabla.(n_i\vec{v}) = n_e \nu_i,\ \ n_i = n_e = n_0 \exp\left(\frac{eU}{k_B T_e}\right) \tag{1}$$

$$p_i = n_i k_B T_i,\ \ \vec{E} = -\nabla U$$

Where ν_i is the ionisation rate, ν_t is the total collision rate, n_0 represents the bulk plasma density and U is the electrostatic potential. It is convenient to use the following dimensionless parameters:

$$X = \frac{x}{\lambda_{mfp}},\ \vec{V} = \frac{\vec{v}}{c_s},\ \phi = -\frac{eU}{k_B T_e},\ \Delta = \frac{\nu_i}{\nu_t} \tag{2}$$

$$c_s = \sqrt{\frac{k_B(T_e + T_i)}{m_i}},\ \lambda_{mfp} = \frac{c_s}{\nu_t},\ \omega\tau = \frac{eB}{m_i \nu_t} \tag{3}$$

Here, c_s is the sound speed, λ_{mfp} is the total collision mean free path and ωτ is the Hall parameter. By invoking the symmetry of the problem, substituting the normalised parameters and separating the vector components one can obtain:

$$V_x \frac{\partial V_y}{\partial X} = \omega\tau \sin(\alpha) V_z - V_y$$
$$V_x \frac{\partial V_z}{\partial X} = \omega\tau \cos(\alpha) V_x - \omega\tau \sin(\alpha) V_y - V_z$$
$$\left(V_x - \frac{1}{V_x}\right)\frac{\partial V_x}{\partial X} = -\omega\tau \cos(\alpha) V_z - \frac{\Delta}{V_x} - V_x \tag{4}$$
$$V_x \frac{\partial \phi}{\partial X} = \Delta - \frac{\partial V_x}{\partial X}$$

This results in three free parameters: Δ, which is the ratio of the ionisation and the total collision rate, ωτ and α (eqn. 2 & 3). The equations of motion (eqn. 4) are integrated numerically using an adaptive step Runge-Kutta method. The integration is cut off when V_x approaches unity. Since the fluid velocities and the potential must be zero in the bulk plasma and given the sheath edge condition the system is overdetermined - the sheath edge position is determined as an eigenvalue. We prefer this to prescribing the presheath width, since the natural length scales are of key interest in this work.

2.2. Profiles

To gauge some of the effects of the magnetic field on the behaviour of the presheath it is helpful to examine how the variables in the equations of motion vary across its width. We consider two different values of ωτ while keeping the field angle constant (α = 20°). We choose the ionisation ratio to be unity (Δ = 1) for all data presented here.

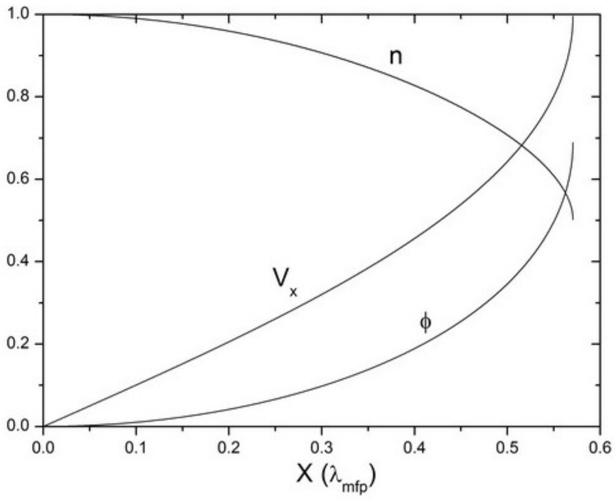

Fig 2: Weakly magnetised presheath profile (ωτ = 0.01, α = 20°); n and φ are the density and normalised potential with respect to the bulk plasma.

Figure 2 shows the presheath for a very low ωτ. It exhibits the well known collision dominated behaviour. One can see that the wall perpendicular fluid velocity increases gradually across the width of the presheath and only begins to diverge when the sheath edge singularity is approached. The cause for this is a non-negligible electric field, which extends over the entire width of the presheath. It accelerates the ions towards the wall unhindered, since there are too many collisions for the ions to be deflected by the magnetic field. At the sheath edge (X_{SE} = 0.57 λ_{mfp}) the potential and density have the normalised values 0.69 and 0.5, respectively. These are the correct values for a collision dominated presheath [3].

This is in stark contrast to figure 3, where the magnetic field is strong enough to prevent cross field flow until the electric force becomes comparable to the magnetic Lorentz force. Effectively, the acceleration required to satisfy the Bohm condition is delayed until close to the sheath edge. We call this region, in which both forces play a major role, the Chodura layer [4]. Due to the fact that the width of this region scales with the Larmor radius (here, we use the Larmor radius of ions travelling at the sound speed), it was necessary to restrict ourselves to a relatively low Hall parameter of 100 in the following figure. An explanation of this particular choice of ωτ will follow in section 4.

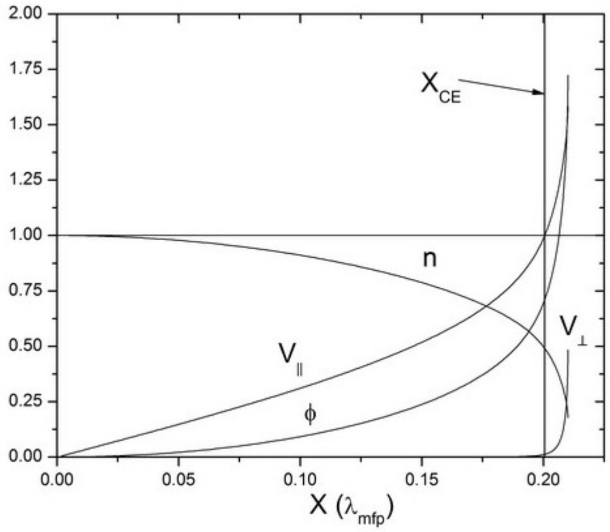

Fig 3: Highly magnetised presheath profile (ωτ = 100, α = 20°); note X_{CE}, the entrance into the Chodura layer. Velocities are defined relative to **B**.

It is interesting to note the behaviour of the presheath before the Chodura edge (CE). Here, we consider the ion fluid velocity components parallel and perpendicular with respect to **B** rather than the wall. The curves of potential and density agree very well with figure 2 once the length scales are matched. $V_\parallel$ takes the place of V_x in determining the boundary of the region, except for the fact that there is no singularity associated with the point where $V_\parallel = 1$ for finite Hall parameter. If we consider the case of $\omega\tau \rightarrow \infty$, we find that the ratio of the Larmor radius to the collision mean free path tends to zero. This means that one would now have to employ a two scale theory akin to that of presheath and sheath [4]. The theoretical basis of the two scale approach can be found in [5].

3. Transition to the Magnetised Regime

As $\omega\tau$ is increased one can identify a transition between two distinct regimes. The plasma parameters at the sheath edge have the well known values of a collision dominated presheath [3] for $\omega\tau < 10^{-1}$. Between $\omega\tau \sim 10^{-1}$ and $\omega\tau \sim 10^2$ a transition occurs. We reach the magnetised regime above $\omega\tau \sim 10^2$, where the sheath edge properties once again become constant (fig. 4). Detailed information on the unmagnetised presheath can be found in [2].

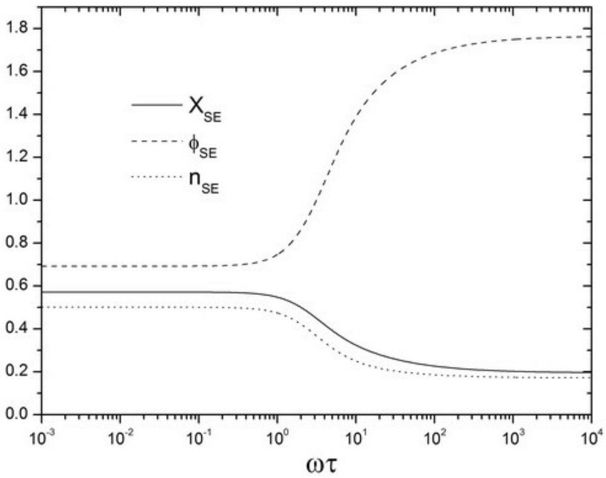

Fig 4: Transition from the collision dominated to the highly magnetised presheath ($\alpha = 20°$).

The collision dominated (low $\omega\tau$) case is completely insensitive to changes in α. Highly magnetised plasmas, on the other hand, have presheath properties which depend strongly on the magnetic field angle (see section 4). Increasing $\omega\tau$ is equivalent to inhibiting cross field transport. This of course requires a greater sheath edge potential to satisfy the Bohm condition and the density adjusts itself accordingly. The width of the whole presheath is decreased in the process.

For high enough magnetisation and intermediate angles it is found that the behaviour of the presheath is very similar to that described by Chodura [4]. The quasineutral presheath separates itself into a layer which is equivalent to the collision dominated presheath, but aligned with the magnetic field, and the Chodura layer. The latter is only separated from the conventional presheath by a singularity in the limit of infinite $\omega\tau$, which is never the case in real plasmas. However, the behaviour of the ion fluid does differ significantly in the two regions, which merits closer investigation. The properties of the Chodura layer and its response to changes in the magnetic field angle are discussed further in the next section. A summary of plasma-wall structures under different conditions is given in [6].

4. Properties of the Magnetised Presheath

The most obvious way to distinguish the **B**-aligned presheath (BAP) from the Chodura layer is by calculating the angle of incidence of the ion flow across the Chodura and sheath edges. One would expect ions to leave the BAP at the magnetic field angle and to be deflected sufficiently at the sheath edge to fulfil the Bohm criterion. However, it should be noted that the formation of a Chodura layer requires that the magnetic field makes an intermediate angle with the wall. For steep angles, deflection from the field is small and the problem is similar to the electrostatic presheath. If α is too small, one approaches a diffusion problem and cross field flow becomes important in the entire presheath.

It is possible to predict how pronounced a Chodura layer one might expect for any given set of parameters. If the flow along the magnetic field, measured at the sheath edge, is highly supersonic, then the Chodura layer will be well defined. Figure 5 shows the sheath edge ion velocity parallel to B for different values of $\omega\tau$ plotted against field angle. It seems that flow along the field is notably supersonic ($> 1.2\ c_s$) for angles between about 5° and 50°. Note that $\omega\tau = 100$ and $\alpha = 20°$, as chosen for the highly magnetised presheath profile (fig. 3), yield highly supersonic ($\sim 1.5\ c_s$) flow at the sheath edge.

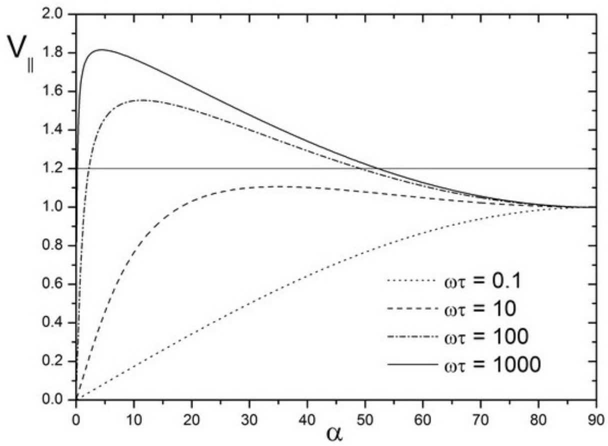

Fig 5: Sheath edge ion velocity parallel to B for different Hall parameters over the entire range of α.

Figure 6 gives a comparison of the angles of incidence of ions into the Chodura layer and into the sheath. It is clear that the ion flow is aligned perfectly at the Chodura edge for most angles, while it is deflected significantly at the sheath edge. The CE alignment curve does not pass through the origin, again indicating that the case of $\alpha \rightarrow 0$ does not produce a Chodura layer.

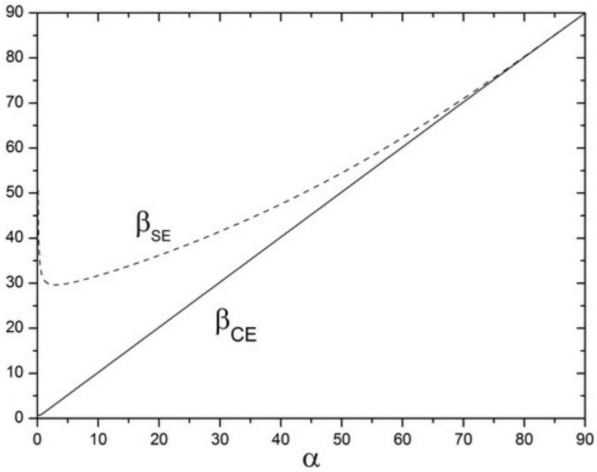

Fig 6: Angle of incidence of the ion fluid into the sheath and the Chodura layer ($\omega\tau = 1000$). The CE curve does not pass through the origin.

Ahedo [6] mentions that the characteristic length scales for the BAP and the Chodura layer are λ_{mfp} and $r_{Li}\cos(\alpha)$, respectively. While these are indeed good estimates, it is possible to give a more precise scale length for the BAP. We find that it is given by the simple expression $X_{CE} = 0.57\ \lambda_{mfp}\sin(\alpha)$ for large Hall parameters and $\alpha > 5°$. Further, the width of the Chodura layer seems to be related to α by a more complicated function than a cosine. Figure 7 shows the BAP width, X_{CE}, normalised to $\lambda_{mfp}\sin(\alpha)$ and the Chodura layer width, $\delta X = X_{SE} - X_{CE}$, normalised to $r_{Li}\cos(\alpha)$.

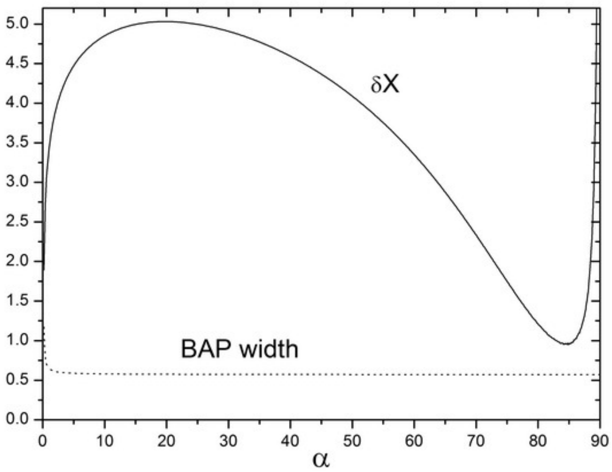

Fig 7: BAP and Chodura layer widths with their scale length estimates as normalisations ($\omega\tau = 1000$).

5. Conclusion

We present a simple fluid model, based on the work of Riemann [1], of the presheath under the influence of a uniform magnetic field at an oblique angle to the wall. It is found that there are two regimes of magnetisation ($\omega\tau < 0.1$, $\omega\tau > 10^2$) in which the sheath edge parameters are insensitive to changes in the Hall parameter. For large $\omega\tau$ ($> 10^2$), profiles of the presheath suggest that the behaviour of the ion fluid can be explained by considering two regions. From the bulk plasma, ions are first constrained to move along the field in what one might call the **B**-aligned presheath (BAP). Near the sheath edge, however, the electric field becomes strong enough to deflect the ions. We refer to this region as the Chodura layer in recognition of the fact that the behaviour is very similar to that outlined in [4]. Such a layer is well defined for angles in the range $50° > \alpha > 5°$. It should be noted that this layer is not separated from the BAP by a singularity, unless $\omega\tau$ becomes infinite.

The model relies on several assumptions, which leave room for generalisation. We assume that the Debye length is far smaller than both the ion Larmor radius and the collision mean free path, thus placing the sheath on a far smaller scale than either presheath layer. For simplicity, the ions are assumed to be isothermal, such that the sound speed is constant. The calculations are also valid when T_i is much less than T_e. The assumption of isothermal ions is then no longer required. Future work could encompass the determination of a more exact Chodura layer scale length and mechanism as well as the adaptation of the model to two- and three-dimensional cases.

6. References

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