Numerical analysis of the potential profile in the sheath formed in front of a floating electron emitting electrode immersed in a two-electron temperature plasma

T. Gyergyek[1,2], M. Čerček[2,3]

[1] University of Ljubljana, Faculty of electrical engineering, Tržaška 25, 1000 Ljubljana, Slovenia
[2] Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia
[3]University of Maribor, Faculty of civil engineering, Smetanova 17, 2000 Maribor, Slovenia

A simple one-dimensional analytical fluid model developed in a recent paper [T. Gyergyek, M. Čerček, Contrib. Plasma Phys., **45**, (2005), 568.] is used to find the floating potential of an electron emitting electrode immersed in a two-electron temperature plasma and the corresponding Bohm velocity of the ions at the sheath entrance. The floating potential is used as a boundary condition for the numerical solution of the Poisson equation. The second boundary condition (the electric field at the collector) is determined so that the numerical solution gives zero potential and electric field at the sheath edge. This can be done only for the largest and the smallest ion velocities at the sheath entrance predicted by the analytical model but not for the intermediate ones.

1. Introduction

In recent papers [1], we have presented our extension of the model developed initially by Takamura and coworkers [2] for the analysis of the emissive probe characteristics. In the next section the model is briefly described and key equations are given without derivation. The derivation can be found in [1]. In section 3 some profiles of the potential and of the electric field in the sheath obtained from the numerical solution of the Poisson equation are shown. Some numerical details are also briefly discussed. In the last section conclusions are given.

2. The model

An infinite plane material surface (collector) located at $x = 0$ in contact with a plasma filling the half-space $x > 0$ is considered. Far from the collector the plasma is quasi-neutral and the potential there is taken as a reference potential, which is set to zero: $\Phi(x\to\infty) = 0$. Also the electric field there is zero. The collector is biased to a certain potential Φ_C, which is negative and smaller (more negative) than $\Phi(x)$ for any $x > 0$. As one approaches the collector the potential slowly decreases. This region of the slow potential drop is called the pre-sheath. There the plasma is still quasi-neutral although a small electric field exists there. This electric field accelerates the positive ions towards the collector and the negative electrons in the opposite direction. The positive ions are assumed to be cold and at rest at a very large distance (beyond the pre-sheath) from the collector. At a certain distance $x = d$ from the collector the plasma potential has a value Φ_S and there the ions reach the velocity v_0 in the direction towards the collector. The plane at $x = d$ is called the sheath edge. At the sheath edge the plasma is still quasi neutral but immediately beyond this point a positive space charge of the sheath exists.

We assume that there are 2 electron populations present in the plasma. Both electron groups have Maxwellian velocity distribution with 2 different temperatures and zero average velocities. We call them the cool and the hot electrons. The collector emits electrons. The emitted electrons leave the collector all with the same initial velocity v_C, which in our model is allowed to be different from zero.

The potential in the sheath $\Phi(x)$ is determined by the Poisson equation:

$$\frac{d^2\Phi}{dx^2} = -\frac{e_0}{\varepsilon_0}\left(n_i(x) - n_1(x) - n_2(x) - n_3(x)\right). \quad (1)$$

Here $n_i(x)$ is the ion density, $n_1(x)$ is the cool electron density, $n_2(x)$ is the hot electron density and $n_3(x)$ is the density of the emitted electrons. The boundary conditions at the sheath edge are:

$$\Phi(x=d) = \Phi_S, \quad \frac{d\Phi}{dx}(x=d) = 0. \quad (2)$$

The second boundary condition is valid because our model is done on the so called sheath scale, which means that $\lambda_D << d << L$, where λ_D is the Deby length defined in (10) below and L is the characteristic length of the pre-sheath. The hot and the cool electrons in the sheath obey the Boltzmann relation:

$$n_1(x) = n_{1S}\exp\left(\frac{e_0\left(\Phi(x)-\Phi_S\right)}{kT_1}\right),\quad n_2(x) = n_{2S}\exp\left(\frac{e_0\left(\Phi(x)-\Phi_S\right)}{kT_2}\right). \tag{3}$$

Here n_{1S} and n_{2S} are the cool and the hot electron densities at the sheath edge, k is the Boltzmann constant, e_0 is the elementary charge, T_1 and T_2 are the cool and the hot electron temperatures. At the sheath edge the density of the ions is n_S and their velocity towards the collector is v_0. From the energy and flux conservation the ion density in the sheath is obtained:

$$n_i(x) = \frac{n_S}{\sqrt{1-\frac{2e_0\left(\Phi(x)-\Phi_S\right)}{m_i v_0^2}}}. \tag{4}$$

The emitted electrons leave the collector all with the same velocity v_C. From the energy and flux conservation their density in the sheath is related to their density at the sheath edge:

$$n_3(x) = \frac{n_{3S}}{\sqrt{1-\frac{2e_0\left(\Phi(x)-\Phi_S\right)}{2e_0\left(\Phi_C-\Phi_S\right)-m_e v_C^2}}}. \tag{5}$$

The fluxes of all the particle species in the sheath are conserved. Their values at the sheath edge are given by:

$$j_1 = e_0 n_{1S}\sqrt{\frac{kT_1}{2\pi m_e}}\exp\left(\frac{e_0\left(\Phi_C-\Phi_S\right)}{kT_1}\right),\quad j_2 = e_0 n_{2S}\sqrt{\frac{kT_2}{2\pi m_e}}\exp\left(\frac{e_0\left(\Phi_C-\Phi_S\right)}{kT_2}\right),\quad j_3 = e_0 n_{3S}\sqrt{v_C^2-\frac{2e_0\left(\Phi_C-\Phi_S\right)}{m_e}},\ j_i = e_0 n_S v_0. \tag{6}$$

In addition the flux of the emitted electrons is related to the fluxes of other particle species by the relation:

$$j_3 = \gamma_i j_i + \gamma\left(j_1 + j_2\right) + j_R. \tag{7}$$

Here γ_i is the secondary emission coefficient for ions, which gives the number of emitted electrons per incident ion and γ is the secondary emission coefficient for electrons. It gives the number of emitted electrons per incident electron and j_R is the Richardson current density of the thermally emitted electrons, which is given by the well known Richardson formula: $j_R = A_R T_C^2\exp(-e_0\Phi_w / kT_C)$. Here A_R is the Richardson constant, T_C is the absolute temperature of the collector and Φ_w is the work function of the collector.

Following the procedure described in [1] the electron densities at the sheath edge n_{1S}, n_{2S} and n_{3S} are expressed in terms of the ion density n_S and inserted into (3) and (5). Then (3), (4) and (5) are inserted into the Poisson equation (1):

$$\frac{d^2\Psi}{dz^2} = \frac{1}{1+\beta_S+G}\cdot\left[\left(1-\frac{J_R+M\gamma_i}{\sqrt{N^2-\frac{2\Psi_C}{\mu}}}\right)\left(\exp\left(\Psi(z)\right)+\beta_S\exp\left(\frac{\Psi(z)}{\Theta}\right)\right)+\left(G+\frac{\left(1+\beta_S\right)\left(J_R+M\gamma_i\right)}{\sqrt{N^2-\frac{2\Psi_C}{\mu}}}\right)\cdot\frac{1}{\sqrt{1-\frac{\Psi(z)}{\Psi_C-\frac{N^2\mu}{2}}}}\right]-\frac{1}{\sqrt{1-\frac{2\Psi(z)}{M}}}, \tag{8}$$

The boundary conditions (2) now become:

$$\Psi\left(z=\frac{d}{\lambda_D}\right)=0,\ \frac{d\Psi}{dz}\left(z=\frac{d}{\lambda_D}\right)=0. \tag{9}$$

The following variables have been introduced:

$$v_0 = M\sqrt{\frac{kT_1}{m_i}},\ v_C = N\sqrt{\frac{kT_1}{m_i}},\ \mu=\frac{m_e}{m_i},\ \Theta=\frac{T_2}{T_1},\ \beta_S=\frac{n_{2S}}{n_{1S}},\ \Psi_C=\frac{e_0\left(\Phi_C-\Phi_S\right)}{kT_1},\ \Psi_S=\frac{e_0\Phi_S}{kT_1},\ J_R=\frac{j_R}{e_0 n_S\sqrt{kT_1/m_i}},\ z=\frac{x}{\lambda_D},\ \lambda_D=\sqrt{\frac{\varepsilon_0 kT_1}{n_S e_0^2}},\ \Psi=\frac{e_0\left(\Phi(x)-\Phi_S\right)}{kT_1},\ G=\frac{\gamma\left(\exp\left(\Psi_C\right)+\beta_S\sqrt{\Theta}\exp\left(\frac{\Psi_C}{\Theta}\right)\right)}{\sqrt{2\pi\left(N^2\mu-2\Psi_C\right)}}. \tag{10}$$

Note that M is not an independent parameter. It is determined by the Bohm criterion, which says that at the sheath entrance the ions must have the ion acoustic velocity. Following the procedure described in [1] the following equation is obtained for M:

$$M=\sqrt{\frac{1+\beta_S+G}{\left(\left(1-\frac{J_R+M\gamma_i}{\sqrt{N^2-\frac{2\Psi_C}{\mu}}}\right)\left(1+\frac{\beta_S}{\Theta}\right)+ +\frac{G+\frac{(1+\beta_S)(J_R+M\gamma_i)}{\sqrt{N^2-\frac{2\Psi_C}{\mu}}}}{2\left(\Psi_C-\frac{N^2\mu}{2}\right)}\right)}}. \quad (11)$$

In the pre-sheat potential drop a larger fraction of the cool than of the hot electrons is reflected. Because of that β_S is larger than the hot to cool electron density ratio β_0 very far (beyond the pre-sheath) away from the collector. Relation between β_0 and β_S is given by [1]:

$$\beta_S=\beta_0\exp\left(\frac{(M^2+2\varphi)(\Theta-1)}{2\Theta}\right). \quad (12)$$

Here φ is the energy lost by the ions because of their collisions with the electrons and neutrals in the pre-sheath normalized to kT_1. When the collector is floating the total current density to the collector is zero:

$$J_t=\frac{1}{1+\beta_S+G}\cdot\left[\left(\frac{1}{\sqrt{2\pi\mu}}- -\frac{J_R+M\gamma_i}{\sqrt{2\pi(N^2\mu-2\Psi_C)}}\right)\left(\exp(\Psi_C)+ +\beta_S\sqrt{\Theta}\exp\left(\frac{\Psi_C}{\Theta}\right)\right)- -\left(G\sqrt{N^2-\frac{2\Psi_C}{\mu}}+(1+\beta_S)(J_R-M\gamma_i)\right)\right]-M=0.$$

(13)

If the electron emission from the collector is increased, eventually the density of the emitted electrons and consequently the negative space charge in front of the collector becomes so high, that the electric field at the collector surface becomes zero. In this case the emission is space charge limited. When this happens the zero electric field condition at the collector is valid:

$$\frac{1}{2}\left(\frac{d\Psi}{dz}\right)^2=\frac{1}{1+\beta_S+G}\cdot\left[\begin{pmatrix}\exp(\Psi_C)-1+\\ +\beta_S\Theta\left(\exp\left(\frac{\Psi_C}{\Theta}\right)-1\right)\end{pmatrix}\cdot\left(1-\frac{J_R+M\gamma_i}{\sqrt{N^2-\frac{2\Psi_C}{\mu}}}\right)+ \cdot +2\cdot\left(G+\frac{(1+\beta_S)(J_R+M\gamma_i)}{\sqrt{N^2-\frac{2\Psi_C}{\mu}}}\right)\cdot\left(\Psi_C-\frac{N^2\mu}{2}\right)\cdot \cdot\left(1-\sqrt{1-\frac{\Psi_C}{\Psi_C-\frac{N^2\mu}{2}}}\right)\right]- -M^2\left(1-\sqrt{1-\frac{2\Psi_C}{M^2}}\right)=0.$$

(14)

3. Results

The equations (11), (13) and (14) are the basic equations of the model. Note that β_S is given by (12) If β_0, Θ, μ, φ, N, γ and γ_i are given, the equations (11), (13) and (14) form a system of 3 equations for 3 unknown quantities: Ψ_C, M and J_R. In some cases this system of equations may have up to 5 solutions. For example when we select β_0 = 0.11, Θ = 50, N = 60, μ = 1/1836, φ = 0.32, γ = γ_i = 0 the system of equations (11), (13) and (14) has 5 solutions which are given in the Table 1:

Table 1: Solutions of the system of equations (11), (13) and (14) for the parameters: β_0 = 0.11, Θ = 50, N = 60, μ = 1/1836, φ = 0.32, γ = γ_i = 0.

M	Ψ_C	J_R
1.2761	-4.02174	22.2677
1.1976	-11.1368	19.29
1.16515	-22.7218	15.309
2.87732	-158.466	1.66944
7.70906	-41.6571	39.5771

In Fig.1 the numerical solutions of the Poisson equation (8) and the corresponding electric fields for the parameters: β_0 = 0.11, Θ = 50, N = 60, μ = 1/1836, φ = 0.32, γ = γ_i = 0 are plotted versus z. The Ψ_C given in the Table 1 is used as a boundary condition and the corresponding M and J_R are inserted into (8) together with the other parameters. The case with M = 2.87732 is not shown in Fig. 1.

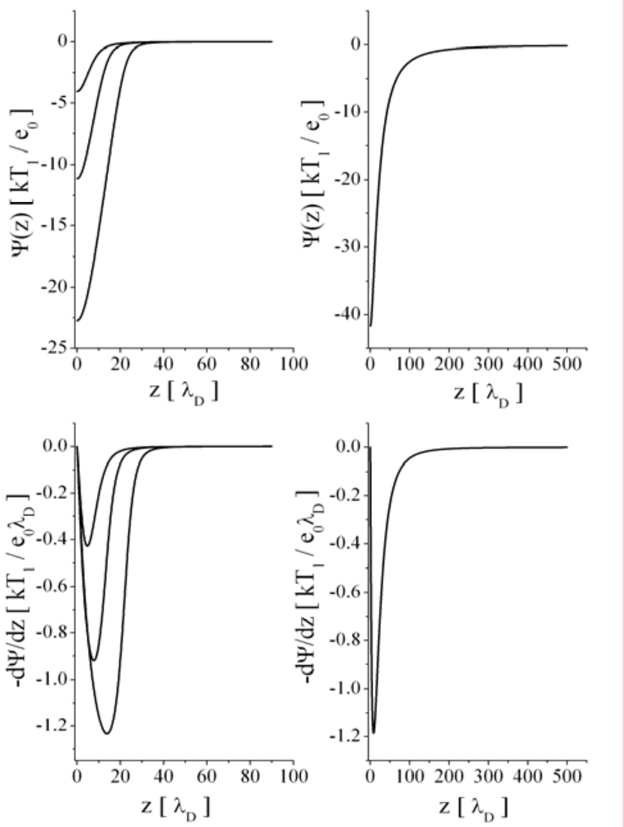

Figure 1: Numerical solutions of the Poisson equation (8) and the corresponding electric fields for the parameters: $\beta_0 = 0.11$, $\Theta = 50$, $N = 60$, $\mu = 1/1836$, $\varphi = 0.32$, $\gamma = \gamma_i = 0$, while M, Ψ_C and J_R are given in Table 1.

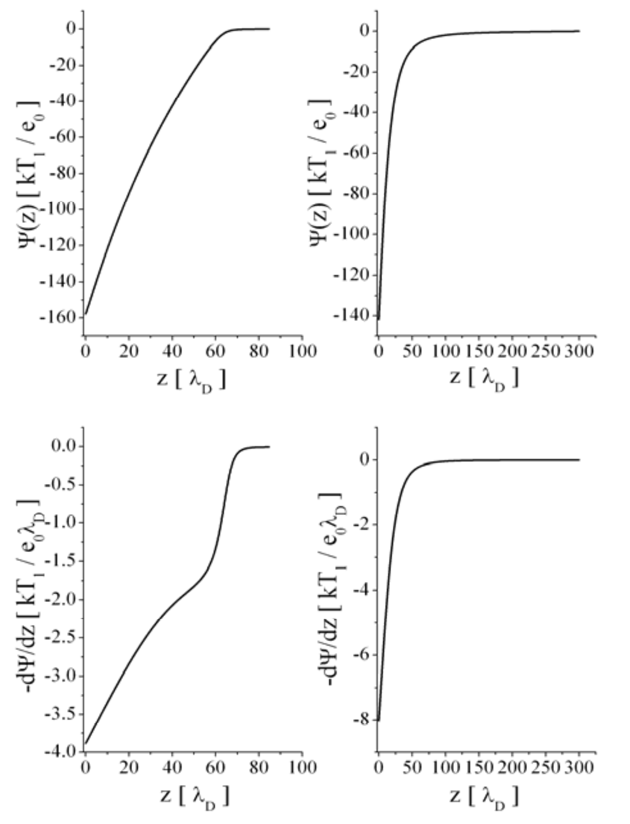

Figure 2: Numerical solutions of the Poisson equation (8) and the corresponding electric fields for the parameters: $\beta_0 = 0.11$, $\Theta = 50$, $N = 60$, $\mu = 1/1836$, $\varphi = 0.32$, $J_R = \gamma = \gamma_i = 0$, while M and Ψ_C are given in Table 2.

Table 2: Solutions of the system of equations (11) and (13) for the parameters: $\beta_0 = 0.11$, $\Theta = 50$, $N = 60$, $\mu = 1/1836$, $\varphi = 0.32$, $J_R = \gamma = \gamma_i = 0$.

M	Ψ_C
1.12863	-157.831
2.8789	-181.448
7.07107	-141.937

The second boundary condition is the value of the electric field $d\Psi/dz$ at $z = 0$. In Fig. 1 this value is always zero, because the emission is space charge limited. In Fig. 2 we show the case, when there is no electron emission from the collector at all. The following parameters are selected: $\beta_0 = 0.11$, $\Theta = 50$, $N = 60$, $\mu = 1/1836$, $\varphi = 0.32$, $J_R = \gamma = \gamma_i = 0$ and the system of equations (11) and (13) is solved for M and Ψ_C. The solutions are given in Table 2. The case with $M = 2.8789$ is not shown in Fig. 2. Note that the smallest M is close to 1, while the largest is close to $\Theta^{1/2}$. The Ψ_C is used as a boundary condition and M is inserted into (8) together with other parameters. The second boundary condition $d\Psi/dz$ at $z = 0$ has to be guessed. This is done with the shooting method.

4. Conclusions

The potential and electric field profiles in the sheath formed in front of a floating electron emitting electrode immersed in a two-electron temperature plasma have been analysed using the numerical solutions of the corresponding Poisson equation. The floating potential found from the theoretical model is used as the first boundary condition. The second boundary condition is the electric field at the collector. In the case of space charge limited emission it is always zero. Otherwise it must be determined in such way that the numerical solution recovers the boundary conditions (9) at the sheath edge. This can always be done for the floating potentials that correspond to M close to 1 and to $\Theta^{1/2}$, but it can not be done for the floating potentials that correspond to the intermediate values of M. The position of the sheath edge can not be predicted by the model, but it is found from the graph of the numerical solution.

References

1) T. Gyergyek, M. Čerček, Contrib. Plasma Phys., **45**, (2005), 89; T. Gyergyek, M. Čerček, Contrib. Plasma Phys., **45**, (2005), 568. 2) M. Y. Ye, S. Takamura, Phys. Plasmas, **7**, (2000), 3457